

LAW WITHOUT LAW

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Abstract

We consider a model for spacetime in which there is an ubiquitous background Dark Energy which is the Zero Point Field. This is further modeled in terms of a Weiner process that leads to a Random or Brownian characterization. Nevertheless we are able to recover meaningful physics, very much in the spirit of Wheeler's Law without Law, that is laws emerging from an underpinning of lawlessness.

1 Introduction

From the beginning of modern science, the universe has been considered to be governed by rigid laws which therefore, in a sense, made the universe somehow deterministic. However, it would be more natural to expect that the underpinning for these laws would be random, unpredictable and spontaneous rather than enforced events. This alternative but historical school of thought is in the spirit of Prigogine's, "Order out of chaos"[1].

Prigogine notes, "As we have already stated, we subscribe to the view that classical science has now reached its limit. One aspect of this transformation is the discovery of the limitations of classical concepts that imply that a knowledge of the world "as it is" was possible. The omniscient beings, Laplace's or Maxwell's demon, or Einstein's God, beings that play such an important role in scientific reasoning, embody the kinds of extrapolation physicists thought they were allowed to make. As randomness, complexity, and irreversibility enter into physics as objects of positive knowledge, we

are moving away from this rather naive assumption of a direct connection between our description of the world and the world itself. Objectivity in theoretical physics takes on a more subtle meaning. ...Still there is only one type of change surviving in dynamics, one "process", and that is motion... It is interesting to compare dynamic change with the atomists' conception of change, which enjoyed considerable favor at the time Newton formulated his laws. Actually, it seems that not only Descartes, Gessendi, and d'Alembert, but even Newton himself believed that collisions between hard atoms were the ultimate, and perhaps the only, sources of changes of motion. Nevertheless, the dynamic and the atomic descriptions differ radically. Indeed, the continuous nature of the acceleration described by the dynamic equations is in sharp contrast with the discontinuous, instantaneous collisions between hard particles. Newton had already noticed that, in contradiction to dynamics, an irreversible loss of motion is involved in each hard collision. The only reversible collision—that is, the only one in agreement with the laws of dynamics—is the "elastic," momentum-conserving collision. But how can the complex property of "elasticity" be applied to atoms that are supposed to be the fundamental elements of nature?

"On the other hand, at a less technical level, the laws of dynamic motion seem to contradict the randomness generally attributed to collisions between atoms. The ancient philosophers had already pointed out that any natural process can be interpreted in many different ways in terms of the motion of and collisions between atoms."

In the words of Wheeler[2], we seek ultimately a "Law without Law." Laws are an apriori blue print within the constraints of which, the universe evolves. The point can be understood in the words of Prigogine [3]

"...This problem is a continuation of the famous controversy between Parmenides and Heraclitus. Parmenides insisted that there is nothing new, that everything was there and will be ever there. This statement is paradoxical because the situation changed before and after he wrote his famous poem. On the other hand, Heraclitus insisted on change. In a sense, after Newton's dynamics, it seemed that Parmenides was right, because Newton's theory is a deterministic theory and time is reversible. Therefore nothing new can appear. On the other hand, philosophers were divided. Many great philosophers shared the views of Parmenides. But since the nineteenth century, since Hegel, Bergson, Heidegger, philosophy took a different point of view. Time is our existential dimension. As you know, we have inherited from the

nineteenth century two different world of views. The world view of dynamics, mechanics and the world view of thermodynamics.”

It may be mentioned that subsequent developments in Quantum Theory, including Quantum Field Theory are in the spirit of the former. Einstein himself believed in this view of what may be called deterministic time - time that is also reversible. On the other hand Heraclitus’s point of view was in the latter spirit. His famous dictum was, ”You never step into the same river twice”, a point of view which was endorsed by earlier ancient Indian thought. This has been the age old tussle between ”being” and ”becoming”.

As Wheeler put it, (loc.cit), ”All of physics in my view, will be seen someday to follow the pattern of thermodynamics and statistical mechanics, of regularity based on chaos, of ”law without law”. Specifically, I believe that everything is built higgledy-piggledy on the unpredictable outcomes of billions upon billions of elementary quantum phenomena, and that the laws and initial conditions of physics arise out of this chaos by the action of a regulating principle, the discovery and proper formulation of which is the number one task....”

The reason this approach is more natural is, that otherwise we would be lead to ask, ”from where have these laws come?” unless we either postulate a priori laws or we take shelter behind an anthropic argument. An interesting but neglected body of work in the past few decades is that of Random or Stochastic Mechanics and Electrodynamics. It may be mentioned that a considerable amount of work has been done in this direction by Nelson, Landau, Prugovecki, the author and others[4]-[30], who have tried to derive the Schrodinger equation, the Klein-Gordon equation and even the Dirac equation from stochastic considerations, and in general develop an underpinning of stochastic mechanics and stochastic electrodynamics. The literature is vast and some of the references given cite an extensive bibliography. A few of these approaches have been very briefly touched upon in Cf.ref.[31]. However, all these derivations contain certain assumptions whose meaning has been unclear. We will see examples of this in the sequel. In any case, we will argue that the seeds of a new world view, of the paradigm shift are to be found here in these considerations.

In the above context, we propose below that purely stochastic processes lead to minimum space-time intervals of the order of the Compton wavelength and time, whose considerable significance will be seen and it is this circumstance that underlies quantum phenomena and cosmology, and, in the

thermodynamic limit in which N , the number of particles in the universe $\rightarrow \infty$, classical phenomena and Quantum Theory as well. In the process, we will obtain a rationale for some of the ad hoc assumptions referred to above. In the older, and more popular world view, spacetime has generally been taken to be a differentiable manifold with an Euclidean (Galilean) or Minkowskian or Riemannian character. Though the Heisenberg Principle in Quantum Theory forbids arbitrarily small space time intervals, the above continuum character with space time points has been taken for granted even in Quantum Field Theory. In fact if we accept the proposition that what we know of the universe is a result of our measurement (which includes our perception), and that measurements are based on quantifiable units, then it becomes apparent that a continuum is at best an idealization. This was the reason behind the paradox of the point electron which was encountered in the Classical theory of the electron, as we saw. It was also encountered as is well known in Dirac's Quantum Mechanical treatment of the relativistic, spinning electron in which the electron showed up with the velocity of light.

Quantum Mechanics has lived with this self contradiction[32]. In this schizophrenic existence, the wave function follows a deterministic (time reversible) equation, while the result of a measurement, without which no information is retrievable, follows from an acausal "collapse of the wave function" yielding one of the many permissible eigen values, in an unpredictable but probabilistic manner. Indeed it has been suggested by Snyder, Lee and others that the infinities which plague Quantum Field Theory are symptomatic of the fact that space time has a granular or discrete rather than continuous character. This has lead to a consideration of extended particles[33]-[39] [40], as against point particles of conventional theory. Wheeler's space time foam and strings[41]-[45] are in this class, with a minimum cut off at the Planck scale. As 't Hooft notes, [46] "It is somewhat puzzling to the present author why the lattice structure of space and time had escaped attention from other investigators up till now..." We will return to this point later.

All this has also lead to a review of the conventional concept of a rigid background space time. More recently [47]-[49], it has been pointed out by the author that it is possible to give a stochastic underpinning to space time and physical laws. This is in the spirit of Wheeler's, "Law without Law" [2] alluded to. In fact in a private communication to the author, Prof. Prigogine wrote, "...I agree with you that spacetime has a stochastic underpinning".

2 The Emergence of Space-Time

We will later briefly survey some models for spacetime. For the moment our starting point is the well known fact that in a random walk, the average distance l covered at a stretch is given by [50]

$$l = R/\sqrt{N} \tag{1}$$

where R is the dimension of the system and N is the total number of steps. We get the same relation in Wheeler's famous travelling salesman problem and similar problems[51] The interesting fact that equation (1) is true in the universe itself with R the radius of the universe $\sim 10^{28}cm$, N the number of the elementary particles in the universe $\sim 10^{80}$ and l the Compton wavelength of the typical elementary particle, for example the pion $\sim 10^{-13}cm$ had been noticed a long time ago[52]. From a different point of view, it is one of the cosmic "coincidences" or Large Number relations, pointed out by Weyl, Eddington and others. In this context, equation (1) which has been generally considered to be accidental (along with other such relations which we will encounter), will be shown to arise quite naturally in a cosmological scheme based on fluctuations. We would like to stress that we encounter the Compton wavelength as an important and fundamental minimum unit of length and will return recurrently to this theme.

It may be mentioned that a minimum time interval, the chronon, has been considered earlier in a different context by several authors as we will see very soon. What distinguishes Quantum Theory from Classical Physics is as pointed out, the role of the resolution of the observer or observing apparatus. What appears smooth at one level of perception, may turn out to be very irregular on a closer examination. Indeed as noted by Abbot and Wise[53], in this respect the situation is similar to everywhere continuous but non differentiable curves, the fractals of Mandelbrot [54]. This again is tied up with the Random Walk or Brownian character of the Quantum path as noted by Sornette and others[55]-[62]: At scales larger than the Compton wavelength but smaller than the de Broglie wavelength, the Quantum paths have the fractal dimension 2 of Brownian paths (cf. also Nottale,[63]). This will be touched upon briefly in Section 6.

This irregular nature of the Quantum Mechanical path was noticed by Feynman [64] "...these irregularities are such that the 'average' square velocity does not exist, where we have used the classical analogue in referring to an

'average'.

"If some average velocity is defined for a short time interval Δt , as, for example, $|x(t + \Delta t) - x(t)|/\Delta t$, the "mean" square value of this is $-\hbar/(m\Delta t)$. That is, the "mean" square value of a velocity averaged over a short time interval is finite, but its value becomes larger as the interval becomes shorter. It appears that quantum-mechanical paths are very irregular. However, these irregularities average out over a reasonable length of time to produce a reasonable drift, or "average" velocity, although for short intervals of time the "average" value of the velocity is very high..."

This as we will see was Dirac's conclusion too, and indeed his explanation for the luminal velocity of the point electron and the non Hermiticity of its position operator in his relativistic electron theory.

Two important characteristics of the Compton wavelength have to be re-emphasized (Cf.[49]): On the one hand with a minimum space time cut off at the Compton wavelength, as we will see, we can recover by a simple coordinate shift the Dirac structure for the equation of the electron, including the spin half. In this sense the spin half, which is purely Quantum Mechanical will be seen to be symptomatic of the minimum space time cut off, as is also suggested by the zitterbewegung interpretation of Dirac (in terms of the Uncertainty Principle), Hestenes and others (Cf. discussion in [31]). The zitterbewegung is symptomatic of the fact that by the Heisenberg Uncertainty Principle, physics begins only after an averaging over the minimum space time intervals. This is also suggested by stochastic models of Quantum Mechanics referred to, both non relativistic and relativistic as also Feynman's Path Integral formulation. We will comment upon in the sequel.

On the other hand, we will see that (1) and a similar equation for the Compton time in terms of the age of the universe, viz.,

$$T \approx \sqrt{N}\tau \tag{2}$$

can be the starting point for a unified scheme for physical interactions and indeed a cosmology that is not only consistent with observation in which we will deduce the Large Number coincidences referred to, but also predicted in 1997 an accelerating expanding universe when the ruling paradigm was exactly the opposite. We will see this in the next Chapter in detail. The Large Number relations also include a mysterious formula [65], connecting the pion mass and the Hubble constant which we will deduce. It has to be pointed out [51] that in the spirit of Wheeler's travelling salesman's "practical man's

minimum" length that the Compton scale plays such a role, and that space time is like Richardson's delineation of a jagged coastline [54] with a thick brush, the thickness of the brush being comparable to the Compton scale. What Richardson found was that the length of the common land boundaries claimed by Portugal and Spain as also Netherlands and Belgium, differed by as much as 20%! The answer to this non-existent border dispute lies in the fact that we are carrying over our concepts of smooth curves or rectifiable arcs to the measurement of real life jagged boundaries or coastlines. As far as these latter are concerned, as Mandelbrot puts it [54] "The result is most peculiar; coastline length turns out to be an elusive notion that slips between the fingers of one who wants to grasp it. All measurement methods ultimately lead to the conclusion that the typical coastline's length is very large and so ill determined that it is best considered infinite...." This is where Hausdorff dimension or the fractal dimension referred to earlier comes in— we are approximating a higher dimensional curve by a one dimensional curve.

Space time, rather than being a smooth continuum, is more like a fractal Brownian curve, what may be called Quantized Fractal Spacetime. All this has been recognized by some scholars, at least in spirit. As V.L. Ginzburg puts it [66] "The special and general relativity theory, non-relativistic quantum mechanics and present theory of quantum fields use the concept of continuous, essentially classical, space and time (a point of spacetime is described by four coordinates $x_l = x, y, z, ct$ which may vary continuously). But is this concept valid always? How can we be sure that on a "small scale" time and space do not become quite different, somehow fragmentized, discrete, quantized? This is by no means a novel question, the first to ask it was, apparently Riemann back in 1854 and it has repeatedly been discussed since that time. For instance, Einstein said in his well known lecture "Geometry and Experience" in 1921: 'It is true that this proposed physical interpretation of geometry breaks down when applied immediately to spaces of submolecular order of magnitude. But nevertheless, even in questions as to the constitution of elementary particles, it retains part of its significance. For even when it is a question of describing the electrical elementary particles constituting matter, the attempt may still be made to ascribe physical meaning to those field concepts which have been physically defined for the purpose of describing the geometrical behavior of bodies which are large as compared with the molecule. Success alone can decide as to the justification of such an

attempt, which postulates physical reality for the fundamental principles of Riemann's geometry outside of the domain of their physical definitions. It might possibly turn out that this extrapolation has no better warrant than the extrapolation of the concept of temperature to parts of a body of molecular order of magnitude'.

"This lucidly formulated question about the limits of applicability of the Riemannian geometry (that is, in fact macroscopic, or classical, geometric concepts) has not yet been answered. As we move to the field of increasingly high energies and, hence to "closer" collisions between various particles the scale of unexplored space regions becomes smaller. Now we may possibly state that the usual space relationships down to the distance of the order of $10^{-15}cm$ are valid, or more exactly, that their application does not lead to inconsistencies. It cannot be ruled out that, the limit is nonexistent but it is much more likely that there exists a fundamental (elementary) length $l_0 \leq 10^{-16} - 10^{-17}cm$ which restricts the possibilities of classical, spatial description. Moreover, it seems reasonable to assume that the fundamental length l_0 is, at least, not less than the gravitational length $l_g = \sqrt{Gh/c^3} \sim 10^{-33}cm$.

"... It is probable that the fundamental length would be a "cut-off" factor which is essential to the current quantum theory: a theory using a fundamental length automatically excludes divergent results".

Einstein himself was aware of this possibility. As he observed [67], "... It has been pointed out that the introduction of a spacetime continuum may be considered as contrary to nature in view of the molecular structure of everything which happens on a small scale. It is maintained that perhaps the success of the Heisenberg method points to a purely algebraic method of description of nature that is to the elimination of continuous functions from physics. Then however, we must also give up, by principle the spacetime continuum. It is not unimaginable that human ingenuity will some day find methods which will make it possible to proceed along such a path. At present however, such a program looks like an attempt to breathe in empty space". To analyse this further, we observe that space time given by R and T of (1) and (2) represents a measure of dispersion in a normal distribution: Indeed if we have a large collection of N events (or steps) of length l or τ , forming a normal distribution, then the dispersion σ is given by precisely the relation (1) or (2).

The significance of this is brought out by the fact that the universe is a

collection of N elementary particles, infact typically pions of size l , as seen above. We consider space time not as an apriori container of these particles but rather as a Gaussian collection of these particles, a Random Heap. At this stage, we do not even need the concept of a continuum.

In this scheme the probability distribution has a width or dispersion $\sim \frac{1}{\sqrt{N}}$ (Cf. ref.[68, 69, 70]), that is the fluctuation (or dispersion) in the number of particles $\sim \sqrt{N}$. This immediately leads to equations (1) and (2).

It must be emphasized that equations (1) and (2) in particular bring out apart from the random feature a holistic or Machian feature in which the large scale universe and the micro world are inextricably tied up, as against the usual reductionist view discussed in detail earlier. This is in fact inescapable if we are to consider a Brownian Heap. This interpretation in which the extent R (or T) in (1) (or (2)) is a dispersion also explains the fractal dimensionality 2: If the steps were laid out one beside the other unidirectionally as in conventional thinking, then we would have the usual dimensionality one. For, instead of (1), we would have,

$$R = Nl$$

This again is tied up with a model in terms of a Weiner process (a Random Walk), as we will see below.

There is another nuance. Newtonian space was a passive container which "contained" matter and interactions - these latter were actors performing on the fixed platform of space. But our view is in the spirit of Liebniz [71] for whom the container of space was made up of the contents - the actors, as it were, made up the stage or platform. This also implies the background independence alluded to earlier, a feature shared by General Relativity.

It should also be observed that the cut off length for fractal behaviour depends on the mass, via the de Broglie or Compton wavelength. The de Broglie wavelength is the non-relativistic version of the Compton wavelength. Indeed as has been shown in detail [72, 73], it is the zitterbewegung or self-interaction effects within the minimum cut off Compton wavelength that give rise to the inertial mass. So the appearance of mass in the minimum cut off Compton (or de Broglie) scale is quite natural. This point will be analyzed further in the sequel.

We can appreciate that the fractal nature and a stochastic underpinning are interrelated: for scales less than the Compton (or de Broglie) wavelength,

time is irregular and can be modelled by a double Wiener process[74]. This will be shown to lead to the complex wave function of Quantum Mechanics, which is one of its distinguishing characteristics (in contrast to Classical theory where complex quantities are a mathematical artifice).

To appreciate all this let us consider the motion of a particle with position given by $x(t)$, subject to random correction given by, as in the usual theory, (Cf.[28, 50, 70]),

$$|\Delta x| = \sqrt{\langle \Delta x^2 \rangle} \approx \nu \sqrt{\Delta t},$$

$$\nu = \hbar/m, \nu \approx lv \quad (3)$$

where ν is the so called diffusion constant and is related to the mean free path l as above. We can then proceed to deduce the Fokker-Planck equation as follows (Cf.[ref.[28] for details):

We first define the forward and backward velocities corresponding to having time going forward and backward (or positive or negative time increments) in the usual manner,

$$\frac{d_+}{dt}x(t) = \mathbf{b}_+, \quad \frac{d_-}{dt}x(t) = \mathbf{b}_- \quad (4)$$

This leads to the Fokker-Planck equations

$$\partial \rho / \partial t + \text{div}(\rho \mathbf{b}_+) = V \Delta \rho,$$

$$\partial \rho / \partial t + \text{div}(\rho \mathbf{b}_-) = -U \Delta \rho \quad (5)$$

defining

$$V = \frac{\mathbf{b}_+ + \mathbf{b}_-}{2} \quad ; U = \frac{\mathbf{b}_+ - \mathbf{b}_-}{2} \quad (6)$$

We get on addition and subtraction of the equations in (5) the equations

$$\partial \rho / \partial t + \text{div}(\rho V) = 0 \quad (7)$$

$$U = \nu \nabla \ln \rho \quad (8)$$

It must be mentioned that V and U are the statistical averages of the respective velocities. We can then introduce the definitions

$$V = 2\nu \nabla S \quad (9)$$

$$V - iU = -2\nu \nabla (\ln \psi) \quad (10)$$

The decomposition of the Schrodinger wave function as

$$\psi = \sqrt{\rho} e^{iS/\hbar}$$

leads to the well known Hamilton-Jacobi type equation

$$\frac{\partial S}{\partial t} = -\frac{1}{2m}(\partial S)^2 + \nu + Q, \quad (11)$$

where

$$Q = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}$$

From (9) and (10) we can finally deduce the usual Schrodinger equation or (11) [74].

We note that in this formulation three conditions are assumed, conditions whose import has not been clear. These are [28]:

(1) The current velocity is irrotational. Thus, there exists a function $S(x, t)$ such that

$$m\vec{V} = \vec{\nabla} S$$

(2) In spite of the fact that the particle is subject to random alterations in its motion there exists a conserved energy, defined in terms of its probability distribution.

(3) The diffusion constant is inversely proportional to the inertial mass of the particle, with the constant of proportionality being a universal constant \hbar (Cf. equation (3)):

$$\nu = \frac{\hbar}{m}$$

We note that the complex feature above disappears if the fractal or non-differential character is not present, (that is, the forward and backward time derivatives(6) are equal): Indeed the fractal dimension 2 also leads to the real coordinate becoming complex. What distinguishes Quantum Mechanics is the adhoc feature, the diffusion constant ν of (3) in Nelson's theory and the "Quantum potential" Q of (11) which appears in Bohm's theory as well, though with a different meaning.

Interestingly from the Uncertainty Principle,

$$m\Delta x \frac{\Delta x}{\Delta t} \sim \hbar$$

we get back equation (3) of Brownian motion. This shows the close connection on the one hand, and provides, on the other hand, a rationale for the particular, otherwise adhoc identification of ν in (3) - its being proportional to \hbar .

We would like to emphasize that we have arrived at the Quantum Mechanical Schrodinger equation from Classical considerations of diffusion, though with some new assumptions. In the above, effectively we have introduced a complex velocity $V - \imath U$ which alternatively means that the real coordinate x goes into a complex coordinate

$$x \rightarrow x + \imath x' \quad (12)$$

To see this in detail, let us rewrite (6) as

$$\frac{dX_r}{dt} = V, \quad \frac{dX_i}{dt} = U, \quad (13)$$

where we have introduced a complex coordinate X with real and imaginary parts X_r and X_i , while at the same time using derivatives with respect to time as in conventional theory.

We can now see from (6) and (13) that

$$W = \frac{d}{dt}(X_r - \imath X_i) \quad (14)$$

That is, in this non relativistic development either we use forward and backward time derivatives and the usual space coordinate as in (6), or we use the derivative with respect to the usual time coordinate but introduce complex space coordinates as in (12).

Let us briefly analyze this aspect though we will return to it later. To bring out the new input here, we will consider the diffusion equation (3) in only one dimension for the moment. We note that through (6) we have introduced a complex velocity W , as indeed can be seen from (13) and (14) as well. Furthermore (8) and (9) show that both U and V can be written as gradients in the form

$$\begin{aligned} \vec{V} &= \vec{\nabla} f \\ \vec{U} &= \vec{\nabla} g \end{aligned} \quad (15)$$

Furthermore the equation of continuity, (7) shows that for nearly constant and homogenous density ρ we have

$$\vec{\nabla} \cdot \vec{V} = 0 \quad (16)$$

where we are still retaining the vector notation. This implies that f and so also g satisfy the Laplacian equation

$$\nabla^2 f = 0 \quad (17)$$

In this case given (17), it is well known from the Theory of Fluid flow [75] that the trajectories $f = \text{constant}$ and $g = \text{constant}$ are orthogonal, with, in the case of spherical symmetry, the former representing radial stream lines and the latter circles around the origin (or more generally closed curves). We also see that (16) shows that the velocity is solenoidal, and \vec{V} being a gradient, by (9), also irrotational. We would then expect that the circulation given by the expression

$$\Gamma = m \oint \vec{V} \cdot d\vec{s} \quad (18)$$

would vanish. All this is true in a simply connected space. However if the space is multiply connected, the origin being the singularity, then the circulation (18) does not vanish. We argue that this is the Quantum Mechanical spin, and will return to this point. But briefly, Γ in (18) equals the Quantum Mechanical spin $\hbar/2$. This follows, if we take the radius of the circuit of integration to be the Compton wavelength \hbar/mc and remember that at this distance, the velocity equals c .

The interesting thing is that starting from a single real coordinate, we have ended up with a complex coordinate, and have characterized thereby, the Quantum Mechanical spin. Indeed as we will shortly see it was noticed by Newman in the derivation of the Kerr-Newman metric, that an inexplicable imaginary shift gives Quantum Mechanical spin. In other words Quantum Mechanics results from a complexification of coordinates, this as can be seen now, being symptomatic of multiply connected spaces, and modelled by the Weiner process above.

Finally, it may be remarked that the original Nelsonian theory itself has been criticized by different scholars [76]-[80].

To get further insight into the foregoing considerations, let us start with the

Langevin equation in the absence of external forces,[50, 81]

$$m \frac{dv}{dt} = -\alpha v + F'(t)$$

where the coefficient of the frictional force is given by Stokes's Law [75]

$$\alpha = 6\pi\eta a$$

η being the coefficient of viscosity, and where we are considering a sphere of radius a . This then leads to two cases.

Case (i):

For t , there is a cut off time τ . It is known (Cf.[50]) that there is a characteristic time constant of the system, given by

$$\frac{m}{\alpha} \sim \frac{m}{\eta a},$$

so that, from Stokes's Law, as

$$\eta = \frac{mc}{a^2} \text{ or } m = \eta \frac{a^2}{c}$$

we get

$$\tau \sim \frac{ma^2}{mca} = \frac{a}{c},$$

that is τ is the Compton time.

The expression for η which follows from the fact that

$$F_x = \eta(\Delta s) \frac{dv}{dz} = m\dot{v} = \eta \frac{a^2}{c} \dot{v},$$

shows that the inertial mass is due to a type of "viscosity" of the background Zero Point Field (ZPF). (Cf. also ref.[82]).

To sum up case (i), if there is a cut off τ , the stochastic formulation leads us back to the minimum space time intervals \sim Compton scale.

To push these small scale considerations further, we have, using the Beckenstein radiation equation[83],

$$t \equiv \tau = \frac{G^2 m^3}{\hbar c^4} = \frac{m}{\eta a} = \frac{a}{c}$$

which gives

$$a = \frac{\hbar}{mc} \quad \text{if} \quad \frac{Gm}{c^2} = a$$

In other words the Compton wavelength equals the Schwarzschild radius, which automatically gives us the Planck mass. Thus as noted the inertial mass is thrown up in these considerations. We will also see that the Planck mass leads to other particle masses.

On the other hand if we work with $t \geq \tau$ we get

$$ac = \frac{2kT}{\eta a}$$

whence

$$kT \sim mc^2,$$

which is the Hagedorn formula for Hadrons[84].

Thus both the Planck scale and the Compton wavelength Hadron scale considerations follow meaningfully.

Case (ii):

If there is no cut off time τ , as is known, we get back, equation (3),

$$\Delta x = \nu \sqrt{\Delta t}$$

and thence Nelson's derivation of the non relativistic Schrodinger equation. We can see here that the absence of a space time cut off leads to the non-relativistic theory, but on the contrary the cut off leads to the origin of the inertial mass (and as we will see, relativity itself). On the other hand, as we saw, the cut off is symptomatic of a multiply connected space- where we cannot shrink circuits to a point.

The relativistic generalization of the above considerations to the Klein-Gordon equation has been even more troublesome[9]. In this case, there are further puzzling features apart from the luminal velocity as in the Dirac equation. For Lorentz invariance, a discrete time is further required. Interestingly, as we will see Snyder had shown that discrete spacetime is compatible with Lorentz transformations. Here again, the Compton wavelength and time cut off will be seen to make the whole picture transparent.

The stochastic derivation of the Dirac equation introduces a further complication. There is a spin reversal with the frequency mc^2/\hbar . This again is

readily explainable in the earlier context of zitterbewegung in terms of the Compton time. Interestingly the resemblance of such a Weiner process to the zitterbewegung of the electron was noticed by Ichinose[85].

Thus in all these cases once we recognize that the Compton wavelength and time are minimum cut off intervals, the obscure or adhoc features become meaningful.

We would like to reiterate that the origin of the Compton wavelength is the random walk equation (1)! One could then argue that the Compton time (or Chronon) automatically follows. This was shown by Hakim [15, 17]. Intuitively, we can see that a discrete space would automatically imply discrete time. For, if Δt could $\rightarrow 0$, then all velocities, $\lim_{\Delta t \rightarrow 0} |\frac{\Delta x}{\Delta t}|$ would $\rightarrow \infty$ as $|\Delta x|$ does not tend to 0! So there would be a minimum time cut off and a maximal velocity and this in conjunction with symmetry considerations can be taken to be the basis of special relativity as we will see below in more detail.

In fact one could show that quantized spacetime is more fundamental than quantized energy and indeed would lead to the latter. To put it simply the frequency is given by c/λ , where λ the wavelength is itself discrete and hence so also is the frequency. One could then deduce Planck's law as will be seen in the next Section (Cf.[86]). This of course, is the starting point of Quantum Theory itself.

At this stage we remark that in the case of the Dirac electron, the point electron has the velocity of light and is subject to zitterbewegung within the Compton wavelength. The thermal wavelength for such a motion is given by

$$\lambda = \sqrt{\frac{\hbar^2}{mkT}} \sim \text{De Broglie wave length}$$

by virtue of the fact that now $kT \sim mv^2$ itself. In the limit $v \rightarrow c$ in the spirit of the luminal velocity of the point Dirac electron or, using the earlier relation, $kT \sim mc^2$, λ becomes the Compton wavelength. To look at this from another point of view, it is known that for a collection of relativistic particles, the various mass centres form a two-dimensional disc perpendicular to the angular momentum vector \vec{L} and with radius (ref.[34])

$$r = \frac{L}{mc} \tag{19}$$

Further if the system has positive energies, then it must have an extension greater than r , while at distances of the order of r we begin to encounter negative energies.

If we consider the system to be a particle of spin or angular momentum $\frac{\hbar}{2}$, then equation (19) gives, $r = \frac{\hbar}{2mc}$. That is we get back the Compton wavelength. Another interesting feature which we will encounter later is the two dimensionality of the space or disc of mass centres.

On the other hand it is known that, if a Dirac particle is represented by a Gaussian packet, then we begin to encounter negative energies precisely at the same Compton wavelength as above. These considerations show the interface between Classical and Quantum considerations.

Infact as has been shown it is this circumstance that leads to inertial mass, while gravitation and electromagnetism (as for example brought out by the Kerr-Newman metric) and indeed QCD interactions also will be seen to follow. In the light of the above remarks, it appears that the fractal or Brownian Heap character of space time is at the root of Quantum behaviour.

3 Spacetime

As remarked in the previous section, the fact that forward and backward time derivatives in the double Wiener process do not cancel leads to a complex velocity (cf.[74]), $V - \imath U$. That is, the usual space coordinate x (in one dimension for simplicity) is replaced by a coordinate like $x + \imath x'$, where x' is a non constant function of time that is, a new imaginary coordinate is introduced. We will now show that it is possible to consistently take $x' = ct$. Let us take the simplest choice for x' , viz., $x' = \lambda t$. Then the imaginary part of the complex velocity in (14) is given by $U = \lambda$. Then we have (cf.[70]),

$$U = \nu \frac{d}{dx}(\ln \rho) = \lambda$$

where ν and ρ have been defined in (3), and in the equation leading to (11). We thus have, $\rho = e^{\gamma x}$, where $\gamma = \lambda/\nu$ and the quantum potential of (11) is given by

$$Q \sim \frac{\hbar^2}{2m} \cdot \gamma^2 \quad (20)$$

In this stochastic formulation with Compton wavelength cut off, it is known that Q turns out to be the inertial energy mc^2 . It then follows from (20) and

the definition of γ , that $\lambda \approx c$.

In other words it is in the above stochastic formulation that we see the emergence of the spacetime coordinates $(x, \imath ct)$ and Special Relativity from a Wiener process in which time is a back and forth process. All this has been in one dimension.

If we now generalize to three spatial dimensions, then as we will see in a moment [87], we get the quaternion formulation with the three Pauli spin matrices replacing \imath , giving the purely Quantum Mechanical spin half of Dirac. On the other hand, the above formulation with minimum space time cut offs will also be shown to lead independently to the Dirac equation. Thus the origin of special relativity, inertial mass and the Quantum Mechanical spin half is the minimum space time cut offs.

We digress for a moment to observe that equations (1) and (2) indicate that the Compton scale is a fundamental unit of space time. We will now show that this quantized space time leads to Planck's quantized energy, as was briefly seen in the previous section.

The derivation is similar to the well known theory[88].

Let the energy be given by

$$E = g(\nu)$$

Then, f the average energy associated with each mode is given by,

$$f = \frac{\sum_{\nu} g(\nu) e^{-g(\nu)/kT}}{\sum_{\nu} e^{-g(\nu)/kT}}$$

Again, as in the usual theory, a comparison with Wien's functional relation, gives,

$$f = \nu F(\nu/kT),$$

whence,

$$E = g(\nu) \propto \nu,$$

which is Planck's law.

Yet another way of looking at it is, as the momentum and frequency of the classical oscillator have discrete spectra so does the energy.

4 Further Considerations

To see all this in greater detail, we observe that if we treat an electron as a Kerr-Newman black hole, then we get the correct Quantum Mechanical $g = 2$ factor, but the horizon of the black hole becomes complex [73, 42].

$$r_+ = \frac{GM}{c^2} + \imath b, b \equiv \left(\frac{GQ^2}{c^4} + a^2 - \frac{G^2 M^2}{c^4} \right)^{1/2} \quad (21)$$

G being the gravitational constant, M the mass and $a \equiv L/Mc$, L being the angular momentum. While (21) exhibits a naked singularity, and as such has no physical meaning, we note that from the realm of Quantum Mechanics the position coordinate for a Dirac particle in conventional theory is given by

$$x = (c^2 p_1 H^{-1} t) + \frac{\imath}{2} c \hbar (\alpha_1 - c p_1 H^{-1}) H^{-1} \quad (22)$$

an expression that is very similar to (21). Infact as was argued in detail [73] the imaginary parts of both (21) and (22) are the same, being of the order of the Compton wavelength.

It is at this stage that a proper physical interpretation begins to emerge. Dirac himself observed as noted, that to interpret (22) meaningfully, it must be remembered that Quantum Mechanical measurements are really averaged over the Compton scale: Within the scale there are the unphysical zitterbewegung effects: for a point electron the velocity equals that of light.

Once such a minimum spacetime scale is invoked, then we have a non commutative geometry as shown by Snyder more than fifty years ago [89, 90]:

$$\begin{aligned} [x, y] &= (\imath a^2 / \hbar) L_z, [t, x] = (\imath a^2 / \hbar c) M_x, etc. \\ [x, p_x] &= \imath \hbar [1 + (a/\hbar)^2 p_x^2]; \end{aligned} \quad (23)$$

The relations (23) are compatible with Special Relativity. Indeed such minimum spacetime models were studied for several decades, precisely to overcome the divergences encountered in Quantum Field Theory [73],[90]-[95], [96, 97].

Before proceeding further, it may be remarked that when the square of a , which we will take to be the Compton wavelength (including the Planck scale, which is a special case of the Compton scale for a Planck mass viz., $10^{-5} gm$), in view of the above comments can be neglected, then we return

to point Quantum Theory.

It is interesting that starting from the Dirac coordinate in (22), we can deduce the non commutative geometry (23), independently. For this we note that the α 's in (22) are given by

$$\vec{\alpha} = \begin{bmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{bmatrix} \quad ,$$

the σ 's being the Pauli matrices. We next observe that the first term on the right hand side is the usual Hermitian position. For the second term which contains α , we can easily verify from the commutation relations of the σ 's that

$$[x_i, x_j] = \beta_{ij} \cdot l^2 \quad (24)$$

where l is the Compton scale.

There is another way of looking at this. Let us consider the one dimensional coordinate in (22) or (21) to be complex. We now try to generalize this complex coordinate to three dimensions. Then as briefly noted, in the previous Section, we encounter a surprise - we end up with not three, but four dimensions,

$$(1, \iota) \rightarrow (I, \sigma),$$

where I is the unit 2×2 matrix. We get the special relativistic Lorentz invariant metric at the same time. (In this sense, as noted by Sachs [87], Hamilton who made this generalization would have hit upon Special Relativity, if he had identified the new fourth coordinate with time).

That is,

$$x + \iota y \rightarrow Ix_1 + \iota x_2 + jx_3 + kx_4,$$

where (ι, j, k) now represent the Pauli matrices; and, further,

$$x_1^2 + x_2^2 + x_3^2 - x_4^2$$

is invariant. Before proceeding further, we remark that special relativistic time emerges above from the generalization of the complex one dimensional space coordinate to three dimensions, just as the relativistic time came out of the one dimensional space coordinate as seen earlier.

While the usual Minkowski four vector transforms as the basis of the four

dimensional representation of the Poincare group, the two dimensional representation of the same group, given by the right hand side in terms of Pauli matrices, obeys the quaternionic algebra of the second rank spinors (Cf.Ref.[98, 99, 87] for details).

To put it briefly, the quaternion number field obeys the group property and this leads to a number system of quadruplets as a minimum extension. In fact one representation of the two dimensional form of the quaternion basis elements is the set of Pauli matrices. Thus a quaternion may be expressed in the form

$$Q = -\imath\sigma_\mu x^\mu = \sigma_0 x^4 - \imath\sigma_1 x^1 - \imath\sigma_2 x^2 - \imath\sigma_3 x^3 = (\sigma_0 x^4 + \imath\vec{\sigma} \cdot \vec{r})$$

This can also be written as

$$Q = -\imath \begin{pmatrix} \imath x^4 + x^3 & x^1 - \imath x^2 \\ x^1 + \imath x^2 & \imath x^4 - x^3 \end{pmatrix}.$$

As can be seen from the above, there is a one to one correspondence between a Minkowski four-vector and Q . The invariant is now given by $Q\bar{Q}$, where \bar{Q} is the complex conjugate of Q .

However, as is well known, there is a lack of spacetime reflection symmetry in this latter formulation. If we require reflection symmetry also, we have to consider the four dimensional representation,

$$(I, \vec{\sigma}) \rightarrow \left[\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \right] \equiv (\Gamma^\mu)$$

(Cf.also.ref. [100] for a detailed discussion). The motivation for such a reflection symmetry is that usual laws of physics, like electromagnetism do indeed show the symmetry.

We at once deduce spin and Special Relativity and the geometry (23) in these considerations. This is a transition that has been long overlooked [101]. It must also be mentioned that spin half itself is relational and refers to three dimensions, to a spin network infact [102, 42]. That is, spin half is not meaningful in a single particle Universe.

While a relation like (24) above has been in use recently, in non commutative models, we would like to stress that it has been overlooked that the origin of this non commutativity lies in the original Dirac coordinates.

The above relation shows on comparison with the position-momentum commutator that the coordinate \vec{x} also behaves like a “momentum”. This can be seen directly from the Dirac theory itself where we have [13]

$$c\vec{\alpha} = \frac{c^2\vec{p}}{H} - \frac{2i}{\hbar}\hat{x}H \quad (25)$$

In (25), the first term is the usual momentum. The second term is the extra “momentum” \vec{p} due to zitterbewegung.

Infact we can easily verify from (25) that

$$\vec{p} = \frac{H^2}{\hbar c^2}\hat{x} \quad (26)$$

where \hat{x} has been defined in (25).

We finally investigate what the angular momentum $\sim \vec{x} \times \vec{p}$ gives - that is, the angular momentum at the Compton scale. We can easily show that

$$(\vec{x} \times \vec{p})_z = \frac{c}{E}(\vec{\alpha} \times \vec{p})_z = \frac{c}{E}(p_2\alpha_1 - p_1\alpha_2) \quad (27)$$

where E is the eigen value of the Hamiltonian operator H . Equation (27) shows that the usual angular momentum but in the context of the minimum Compton scale cut off, leads to the “mysterious” Quantum Mechanical spin. In the above considerations, we started with the Dirac equation and deduced the underlying non commutative geometry of spacetime. Interestingly, starting with Snyder’s non commutative geometry, based solely on Lorentz invariance and a minimum spacetime length, which we have taken to be the Compton scale, (23), it is possible to deduce the relations (27), (26) and the Dirac equation itself as we will see later.

We have thus established the correspondence between considerations starting from the Dirac theory of the electron and Snyder’s (and subsequent) approaches based on a minimum spacetime interval and Lorentz covariance. It can be argued from an alternative point of view that Special Relativity operates outside the Compton wavelength as we saw earlier.

We started with the Kerr-Newman black hole. Infact the derivation of the Kerr-Newman black hole itself begins with a Quantum Mechanical spin yielding complex shift, which Newman has found inexplicable even after several decades [103, 104]. As he observed, ”...one does not understand why it works.

After many years of study I have come to the conclusion that it works simply by accident". And again, "Notice that the magnetic moment $\mu = ea$ can be thought of as the imaginary part of the charge times the displacement of the charge into the complex region... We can think of the source as having a complex center of charge and that the magnetic moment is the moment of charge about the center of charge... In other words the total complex angular momentum vanishes around any point z^a on the complex world-line. From this complex point of view the spin angular momentum is identical to orbital, arising from an imaginary shift of origin rather than a real one... If one again considers the particle to be "localized" in the sense that the complex center of charge coincides with the complex center of mass, one again obtains the Dirac gyromagnetic ratio..."

The unanswered question has been, why does a complex shift somehow represent spin about that axis? The question has now been answered. Complexified spacetime is symptomatic of fuzzy spacetime and a non commutative geometry and Quantum Mechanical spin and relativity. Indeed Zakrzewski has shown in a classical context that non commutativity implies spin [105, 106]. We will return to these considerations later.

The above considerations recovered the Quantum Mechanical spin together with classical relativity, though the price to pay for this was minimum space-time intervals and noncommutative geometry.

5 The Path Integral Formulation

We come to another description of Quantum Mechanics and first argue that the alternative Feynman Path Integral formulation essentially throws up fuzzy spacetime. To recapitulate [64, 107, 108], if a path is given by

$$x = x(t)$$

then the probability amplitude is given by

$$\phi(x) = e^{i \int_{t_1}^{t_2} L(x, \dot{x}) dt}$$

So the total probability amplitude is given by

$$\sum_{x(t)} \phi(x) = \sum e^{i \int_{t_1}^{t_2} L(x, \dot{x}) dt} \equiv \sum e^{\frac{i}{\hbar} S}$$

In the Feynman analysis, the path

$$x = \bar{x}(t)$$

appears as the actual path for which the action is stationery. From a physical point of view, for paths very close to this, there is constructive interference, whereas for paths away from this the interference is destructive.

We will see later that this is in the spirit of the formulation of the random phase. However it is well known that the convergence of the integrals requires the Lipschitz condition viz.,

$$\Delta x^2 \approx a \Delta t \tag{28}$$

We could say that only those paths satisfying (28) constructively interfere. We would now like to observe that (28) is the same as the Brownian or Diffusion equation (3) related to our earlier discussion of the Weiner process. The point is that (28) again implies a minimum spacetime cut off, as indeed was noted by Feynman himself [64], for if Δt could $\rightarrow 0$, then the velocity would $\rightarrow \infty$.

To put it another way we are taking averages over an interval Δt , within which there are unphysical processes as noted. It is only after the average is taken, that we recover physical spacetime intervals which hide the fractality or unphysical feature. If in the above, Δt is taken as the Compton time (and a is identified with the earlier ν , then we recover for the root mean squared velocity, the velocity of light.

As we have argued in detail this is exactly the situation which we encounter in the Dirac theory of the electron. There we have the unphysical zitterbewegung effects within the Compton time Δt and as $\Delta t \rightarrow 0$ the velocity of the electron tends to the maximum possible velocity, that of light [109]. It is only after averaging over the Compton scale that we recover meaningful physics.

This existence of a minimum spacetime scale, it has been argued is the origin of fuzzy spacetime, described by a noncommutative geometry, consistent with Lorentz invariance viz., equations (23) and (24).

We reiterate that the momentum position commutation relations lead to the usual Quantum Mechanical commutation relations in the usual (commutative) spacetime if $O(l^2)$ is neglected where l defines the minimum scale. Indeed, we have at the smallest scale, a quantum of area reflecting the fractal dimension, the Quantum Mechanical path having the fractal dimension

2 (Cf.ref.[53]). It is this “fine structure” of spacetime which is expressed in the noncommutative structure (23) or (24). Neglecting $O(l^2)$ is equivalent to neglecting the above and returning to usual spacetime. In other words Snyder’s purely classical considerations at a Compton scale lead to Quantum Mechanics.

In the light of the above comments, we can now notice that within the Compton time, we have a double Wiener process leading to non differentiability with respect to time. That is, at this level time in our usual sense does not exist. To put it another way, within the Compton scale we have the complex or non-Hermitian position coordinates for the Dirac electron and zitterbewegung effects - these are unphysical, non local and chaotic in a literal sense. This is a Quantum Mechanical and an experimental fact. It expresses the Heisenberg Uncertainty Principle - space time points imply infinite momenta and energies and are thus not meaningful physically. However as noted earlier Quantum Theory has lived with this contradiction. To put it simply to measure space or time intervals we need units which can be to a certain extent and not indefinitely subdivided - but already this is the origin of discreteness. That is, our measurements are resolution dependent. So physical time emerges at values greater than the minimum unit, which has been shown to be at the Compton scale. Going to the limit of space-time points leads to the well known infinities of Quantum Field Theory (and classical electron theory) which require renormalization for their removal.

The conceptual point here is that time is in a sense synonymous with change, but this change has to be tractable or physical. The non differentiability with respect to time, symbolized and modeled by the double Wiener process, within the Compton time, precisely highlights time or change which is not tractable, that is is unphysical. However Physics, tractability and differentiability emerge from this indeterminism once averages over the zitterbewegung or Compton scale are taken. It is now possible to track time physically in terms of multiples of the Compton scale.

6 Discussion

1. We would like to make the following observations:
 - i) We have in effect equated the statistical fluctuations, when there are N

particles to the Quantum Mechanical fluctuations. The former fluctuations take place over a scale $\sim R/\sqrt{N}$, where R is the size of the system of particles and N is the number of particles in the system. The Quantum Mechanical fluctuations take place at a scale of the order of the Compton wavelength. Apart from the fact that the equality of these two has been taken to be an empirical coincidence, we actually deduce this equality in our cosmology in the next Chapter. Thus the equality is no longer accidental or ad hoc. However a nuance must be borne in mind. In the conventional theory, the Quantum Mechanical fluctuation is a reductionist effect, whereas the statistical fluctuation is a "thermodynamic" or statistical effect in a collection of particles.

ii) In the random mechanical approach, including Nelson's, we encounter "potential" Q - this represents in the usual theory a peculiar correlation between the random motion of a particle and its probability distribution function.

iii) We would like to point out that it would be reasonable to expect that the Weiner process discussed earlier is related to the ZPF which is the Zero Point Energy of a Quantum Harmonic oscillator. We can justify this expectation as follows: Let us denote the forward and backward time derivatives as before by d_+ and d_- . In usual theory where time is differentiable, these two are equal, but we have on the contrary taken them to be unequal. Let

$$d_- = a - d_+ \quad (29)$$

Then we have from Newton's second law in the absence of forces,

$$\ddot{x} + k^2 x = a\dot{x} \quad (30)$$

wherein the new nondifferentiable effect (29) is brought up. In a normal vacuum with usual derivatives and no external forces, Newtonian Mechanics would give us instead the equation

$$\ddot{x} = 0 \quad (31)$$

A comparison of (30) and (31) shows that the Weiner process converts a uniformly moving particle, or a particle at rest into an oscillator. Indeed in (30) if we take as a first approximation

$$\dot{x} \approx \langle \dot{x} \rangle = 0 \quad (32)$$

then we would get the exact oscillator equation

$$\ddot{x} + k^2 x = 0 \quad (33)$$

for which in any case, consistently (32) is correct. We can push these considerations even further and deduce alternatively, the Schrodinger equation, as seen earlier. The genesis of Special Relativity too can be found in the Weiner process. Let us examine this more closely.

We first define a complete set of base states by the subscript i and $U(t_2, t_1)$ the time elapse operator that denotes the passage of time between instants t_1 and t_2 , t_2 greater than t_1 . We denote by, $C_i(t) \equiv \langle i | \psi(t) \rangle$, the amplitude for the state $|\psi(t)\rangle$ to be in the state $|i\rangle$ at time t , and [72, 73]

$$\langle i | U | j \rangle \equiv U_{ij}, U_{ij}(t + \Delta t, t) \equiv \delta_{ij} - \frac{i}{\hbar} H_{ij}(t) \Delta t.$$

We can now deduce from the super position of states principle that,

$$C_i(t + \Delta t) = \sum_j [\delta_{ij} - \frac{i}{\hbar} H_{ij}(t) \Delta t] C_j(t) \quad (34)$$

and finally, in the limit,

$$i\hbar \frac{dC_i(t)}{dt} = \sum_j H_{ij}(t) C_j(t) \quad (35)$$

where the matrix $H_{ij}(t)$ is identified with the Hamiltonian operator. We have argued earlier at length that (35) leads to the Schrodinger equation [72, 73]. In the above we have taken the usual unidirectional time to deduce a non relativistic Schrodinger equation. If however we consider a Weiner process in (34) then we will have to consider instead of (35)

$$C_i(t - \Delta t) - C_i(t + \Delta t) = \sum_j \left[\delta_{ij} - \frac{i}{\hbar} H_{ij}(t) \right] C_j^{(t)} \quad (36)$$

Equation (36) in the limit can be seen to lead to the relativistic Klein-Gordon equation rather than the Schrodinger equation [110]. This is an alternative justification for our earlier result that Special Relativity emerges from the above considerations.

2. We have seen that the path integral formulation is an alternative to the

Schrodinger equation, an alternative that has a resemblance to the stochastic mechanics encountered earlier. However we should bear in mind that these paths are merely mathematical tools for computing the evolution of the wave functions [111]. Nevertheless we should note that the path integral formulation does not give the probability distribution on the space of all paths, so that we cannot legitimately conclude that nature chooses one of the several paths at random according to the probability distribution. Unfortunately in this formulation the measure is complex and not even rigorously defined in the limit of the continuum. Nor will the imaginary or real parts of the measure give the actual Quantum Mechanical picture. It would be more correct to say that the paths are possible paths for a part of the Quantum Mechanical wave. In any case, all this reflects via (28), the unphysicality within the minimum interval Δt .

On the other hand there is the well known Bohmian formulation of Quantum Mechanics which uses the Schrodinger wave function, and the Schrodinger equation to deduce the Hamilton-Jacobi equation exactly as in the stochastic case. But the resemblance is superficial. This non relativistic formulation is one in which the observer plays no part. There is a hidden variable in the form of the position coordinate of the particle. Thus one of the Bohmian paths represents the actual motion of the particle, which exists separately from the wave function. Moreover the Quantum potential Q in the Bohmian case has a non local character and no clear explanation. Furthermore there is no clear generalization to the relativistic case. For all these reasons though Bohm studied this approach in the 1950s, it has not really caught on and we will not pursue the matter further.

3. As mentioned discrete space time and some of their effects have been studied from different points of view for several decades now. It is worth mentioning here that the usual notion of time as an operator with continuous eigen values in Quantum Theory runs into difficulty, as was appreciated by Pauli a long time ago[112]. This can be seen by a simple argument, and, we follow Park[113]: Let the time operator be denoted by \hat{T} , satisfying

$$[\hat{T}, \hat{H}] = i.$$

Let $|E' >$ be an eigenfunction of \hat{H} belonging to the eigenvalue E' , and let $|E' >_{\epsilon} = e^{i\epsilon\hat{T}}|E' >$. Then

$$\hat{H}|E' >_{\epsilon} = e^{i\epsilon\hat{T}}e^{-i\epsilon\hat{T}}\hat{H}e^{i\epsilon\hat{T}}|E' > = (E' + \epsilon)|E' > \quad (37)$$

Remembering that ϵ is arbitrary, (37) gives a continuous energy spectrum, contrary to Quantum Theory. The difficulty is resolved if in the above considerations time were discrete.

4. It must be emphasized that in the stochastic formulation given in this Chapter, there are no hidden variables as in the Bohm formulation, due to the randomness or stochasticity, itself[63].

5. Though we will return to some of the above considerations later, it must be re-emphasized that in the absence of the double Wiener process alluded to, the imaginary part of the complex velocity potential U , vanishes, that is, so does ν of equation (3). In this case we come back to the domain of classical non-relativistic physics. So the origin of special relativity and Quantum Mechanics is to be found here in this double Wiener process within the Compton scale [31]. As pointed out in [73] non-relativistic Quantum Mechanics is not really compatible with Galilean or Newtonian Mechanics.

6. Finally, we would like to reemphasize the following point: By neglecting terms of the order l^2 (the squared Compton length), we return to point, commutative space time and can still have Quantum Mechanics and even relativistic Quantum Mechanics and Quantum Field Theory, though we would then have to introduce Quantum Mechanical spin by separate arguments and consider averages over the Compton scale anyway. But in the process, we are neglecting the Quantum of area or Abbot and Wise's fractal dimension of the Quantum Mechanical path. That is, we are snuffing out the fine structure implied by Quantum Theory and are then using, as remarked earlier, a thick brush to fudge. A quick way to see the result of Abbot and Wise is as follows [63]. From (3) it follows that

$$\langle v^2 \rangle \propto (\Delta t)^{-1}$$

Now if the Hausdorff dimension [54] is D , we would have,

$$\Delta t = (\Delta x)^D$$

whence

$$\langle v^2 \rangle \propto (\Delta t)^{2[(\frac{1}{D})-1]}$$

A comparison yields, $D = 2$.

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